KLM quantum computation with bosonic atoms

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A Knill-Laflamme-Milburn (KLM) type quantum computation with bosonic neutral atoms or bosonic ions is suggested. Crucially, as opposite to other quantum computation schemes involving atoms (ions), no controlled interactions between atoms (ions) involving their internal levels are required. Versus photonic KLM computation, this scheme has the advantage that single atom (ion) sources are more natural than single photon sources, and single atom (ion) detectors are far more efficient than single photon ones.

The purpose of this short note is to point out that one can perform Knill, Laflamme and Millburn (KLM) type quantum computation [1] not only with photons but also with bosonic neutral atoms or with bosonic ions. Such a scheme has a number of practical advantages both over optical KLM computation as well as over traditional quantum computation using neutral atoms/ions.

At present there are many proposals for performing quantum computation with both neutral atoms as well as with ions. All these proposals require carefully controlled interactions between the neutral atoms (ions) and involve manipulating their internal states. In contrast, the KLM type computation presented below does not involve any interaction between the internal degree of freedoms of the atoms(ions). In fact all interactions are actually avoided! The computation involves only manipulating their center of mass.

Conceptually, KLM computation with neutral bosonic atoms and bosonic ions are identical, so, to simplify terminology, in what follows I will use the word "atoms" to represent both neutral bosonic atoms and bosonic ions. It is however worthwhile mentioning right from the beginning that due to the much stronger interaction that ions have with each other and with the environment KLM computation with ions is probably impractical.

One of the major stumbling blocks in KLM quantum computation is the need for deterministic single photon sources which act as the input state for the computation. Laser pulses, that are easy to produce on demand, are not single photon states - they are coherent states, i.e. they are in a superposition of different photon numbers. One way to prepare single photon states is by parametric down-conversion, a process in which one ultra-violet photon impinging on a suitable crystal has a probability of being converted into a pair of optical photons. Detecting one of the photons in a pair guarantees that one optical photon (its partner) is present. However, this process is probabilistic, and gives only a-posteriory information that a state containing one optical photon has been produced. True single photon deterministic sources have already been constructed [2] but are at the limit of present day technology, and generating in a synchonised way a large number of single photon states seems quite remote. On the other hand, by their very nature, atoms are always found in single (or well-determined number) states, so preparing the input state should be much easier.

A second major practical problem in photon KLM computation is the need for single-photon detectors which perform the measurement of the final and certain intermediate states. Such detectors are crucial for driving the computation (by feed forward). But single photon detectors are notoriously inefficient; they are subjected both to losses (when a photon reaches the detector but the detector doesn't click) as well as to "dark counts" (when no photon is present but the detector nevertheless clicks). On the other hand, detecting atoms with high efficiency is rather easy.

A KLM computation with bosonic atoms should take place in a very similar way to a computation with photons. Whenever in a photon computation we inject a photon, we now shoot an atom. Optical mirrors and beamsplitters are replaced with their equivalents for atoms, and can be realized by appropriate arrangements of electric fields or laser beams.

As a technicality, to avoid instabilities affecting the trajectory of the atoms as they travel through the complex multi-particle interferometer that is a KLM computer, one can imagine that individual atoms are captured in the ground-state of individual potential wells, and then the potential wells are moved and drag the captured atoms with them. Of course, there will be instances when we do not know where an atom is since the state can be a superposition of the atom being in many different locations. This happens, for example, after an atom impinges onto a beam-splitter. To insure the guiding effect we must use a moving potential well associated to each incoming/outgoing mode (see fig. 1). The atom will then end up in a superposition of being in different potential wells.

There are already various practical ways of guiding both neutral atoms as well as ions and the technology is improving very fast.

At first sight it seems that KLM computation with atoms cannot work because KLM computation implies "linear" interactions. More precisely, photons do not scatter when they meet, and at each element (mirror, beamsplitter, etc.) each photon behaves as if other photons are not present. For example if say, a piece of appa-

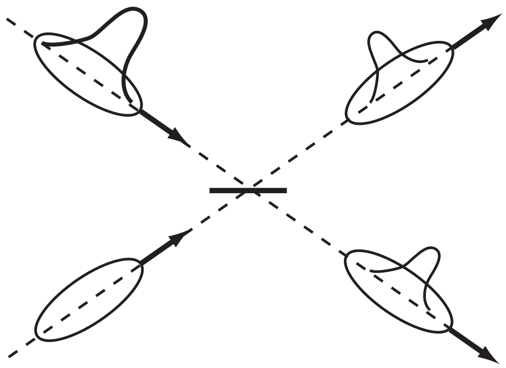

FIG. 1: The diagram shows an atomic wavepacket impinging onto a beamsplitter guided by moving potential wells. Note that there is one potential well per mode.

ratus makes a single photon state evolve according to

$$a^\dagger|0>\rightarrow b^\dagger|0>, \tag{1}$$

then an n photon state evolves according to

$$(a^\dagger)^n|0>\rightarrow (b^\dagger)^n|0> \tag{2}$$

On the other hand, unlike photons, atoms interact with each other and scatter each other. Hence one might expect that the evolution of atoms in a KLM computer will, in general, be completely different from that of photons. I will argue however that this need not be a problem and that one can engineer a regime in which an atom KLM interferometer works almost identically to a photon KLM interferometer.

The main effect in KLM computation is the interference that occurs when two photons impinge simultaneously on a beamsplitter, one photon from each side. Let $a_1^\dagger$ and $a_2^\dagger$ denote the incoming modes and $b_1^\dagger$ and $b_2^\dagger$ denote the outgoing modes. Suppose first that a single photon impinges on the beamsplitter (considered to be 50%-50%), in mode $a_1^\dagger$. The standard evolution is

$$a_1^\dagger|0>\rightarrow \frac{1}{\sqrt{2}}(b_1^\dagger+b_2^\dagger)|0>. \tag{3}$$

On the other hand, when a single photon in mode $a_2^\dagger$ impinges on the beamsplitter, the evolution is

$$a_2^\dagger|0>\rightarrow \frac{1}{\sqrt{2}}(b_1^\dagger-b_2^\dagger)|0>. \tag{4}$$

Now, when two photons impinge together on the beamsplitter, the evolution is

$$a_2^\dagger a_1^\dagger|0>\rightarrow \frac{1}{\sqrt{2}}(b_1^\dagger-b_2^\dagger)|\frac{1}{\sqrt{2}}(b_1^\dagger+b_2^\dagger)|0> \tag{5}$$

$$=\frac{1}{2}\left((b_1^\dagger)^2-(b_2^\dagger)^2\right)|0>. \tag{6}$$

Hence, the photons emerging from the beamsplitter are correlated, in a superposition of both being in mode $b_1^\dagger$ or both in mode $b_2^\dagger$, that is, both leave the beamsplitter in the same direction. (In quantum optics this effect is known as the Hong-Ou-Mandel dip[3]).

Suppose however that we are dealing with atoms instead of photons. When a single atom impinges on an (atomic) beamsplitter, the evolution is similar to that described in equations (3) and (4). On the other hand, when two atoms reach the beamsplitter simultaneously they will colide and scatter so that instead of the interference effect described in (6), we expect the evolution to be described by

$$a_2^\dagger a_1^\dagger|0>\rightarrow |scattering\ state>. \tag{7}$$

The atoms will not emerge both in the same mode but may end up one on each side of the beamsplitter, or scattered in other directions altogether. Since the Hong-Ou-Mandel dip interference is the basic effect that makes KLM computation work, it seems that there is no way to implement it with atoms.

The situation however is not so bad. To see this we need to look more carefully at the incoming wave-packets, and analyze in detail how the scattering occurs. Suppose that the incoming modes $a_1^\dagger$ and $a_2^\dagger$ represent wave-packets that are much longer than the scattering cross-section of the atoms. Let us decompose these long wave-packets into shorter wave-packets, each short wave-packet having the dimension of the scattering length. Let these short wave-packets be described by $a_{1,t_i}^\dagger$ and $a_{2,t_i}^\dagger$, and the corresponding outgoing modes be described by $b_{1,t_i}^\dagger$ and $b_{2,t_i}^\dagger$, . The index t_i may be thought of indicating the time the wave-packet reaches the beamsplitter. Then

$$a_1^\dagger=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}a_{1,t_i}^\dagger \tag{8}$$

$$a_2^\dagger=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}a_{2,t_i}^\dagger, \tag{9}$$

where $n=\frac{L}{l}$ represents the number of short wave-packets (length l) that make the long wave-packet (length L).

Suppose now that instead of the long wave-packets $a_1^\dagger$ and $a_2^\dagger$ we send towards the beamsplitter only two short wavepackets, $a_{1,t_i}^\dagger$ and $a_{2,t_j}^\dagger$. If $i=j$, the two wave-packets arrive at the beamsplitter simultaneously and the atoms scatter

$$a_{2,t_i}^\dagger a_{2,t_i}^\dagger|0>\rightarrow |scattered\ state_i>. \tag{10}$$

On the other hand, when $i\neq j$ the atoms arive at the beamsplitter at different times. Since they arive such

that the distance between them is larger than the scattering lengt, they do not disturb each other and they evolve independently, in he same way as photons would do, namely

$$a^\dagger_{2,t_j}a^\dagger_{2,t_i}|0> \rightarrow \frac{1}{\sqrt{2}}(b^\dagger_{1,t_j}-b^\dagger_{2,t_j})\frac{1}{\sqrt{2}}(b^\dagger_{1,t_i}+b^\dagger_{2,t_i})|0> \tag{11}$$

Hence, out of the n^2 orthogonal, equal magnitude terms in the original state $a^\dagger_2 a^\dagger_1|0>= \frac{1}{\sqrt{n}}\sum_{j=1}^n a^\dagger_{2,t_j}\frac{1}{\sqrt{n}}\sum_{i=1}^n a^\dagger_{1,t_i}|0>$ only n equal magnitude, orthogonal terms (the terms in which $i=j$) lead to scattering, while all others behave as in the photon case. Therefore, in the case of long (relative to the scattering lengths) wave packets, atoms behave identically to photons, up to corrections of $1/n$. That is, instead of (7), two atoms in long wave packets, impinging on the two sides of a beamsplitter actually lead to an Hong-Ou-Mandel dip,

$$a^\dagger_2 a^\dagger_1|0> \rightarrow \frac{1}{2}\big((b^\dagger_1)^2-(b^\dagger_2)^2\big)|0> + O(1/n) \tag{12}$$

where $O(1/n)$ denotes corrections to the wavefunction with norm of the order $1/n$.

A similar analysis can be done for the case of more atoms impinging on a beam-splitter. As long as the wave-packets are long relative to the scattering length, scattering is very unlikely to occur, and atoms behave identically to photons. Hence, in this regime, throughout the whole KLM computer, atoms behave like photons, up to corrections that can be made as small as we want by enlarging the size of the wave-packets. Furthermore, note that by construction the original photonic KLM computation already had to accept errors (the gates are probabilistic); the errors introduced by scattering in the case of atom KLM computation can be dealt with in similar ways to the original errors.

In practice, the cross-section for neutral atom scattering is of the order of angstroms (10^{-10}m), so it is conceivable that atoms can be prepared in long enough wave-packets to make the total amount of errors small enough.

In conclusion, in this note I argued that KLM quantum computation can be performed not only with photons, as originally envisaged, but also with bosonic neutral atoms or bosonic ions. Versus other quantum computation schemes with neutral atoms or ions, KLM computation has the advantage that no controlled interaction between their internal levels are required - only control over the center of mass movement is needed. Versus photonic KLM computation, this scheme has the advantage that single atom/ion sources are more natural than single photon sources, and the detectors are far more efficient than single photon detectors. At the same time, the neutral atom/ion "optics" required for this scheme is far less developed that standard photonic optics, so it is not at all clear which scheme is more advantageous in the long run. Ultimately, all kinds of details of technology will have the decisive role.

Acknowledgments

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